

On Perturbations in Warm Inflation

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Warm inflation is an interesting possibility of describing the early universe, whose basic feature is the absence, at least in principle, of a preheating or reheating phase. Here we analyze the dynamics of warm inflation generalizing the usual slow-roll parameters that are useful for characterizing the inflationary phase. We study the evolution of entropy and adiabatic perturbations, where the main result is that for a very small amount of dissipation the entropy perturbations can be neglected and the purely adiabatic perturbations will be responsible for the primordial spectrum of inhomogeneities. Taking into account the COBE-DMR data of the cosmic microwave background anisotropy as well as the fact that the interval of inflation for which the scales of astrophysical interest cross outside the Hubble radius is about 50 e-folds before the end of inflation, we could estimate the magnitude of the dissipation term. It was also possible to show that at the end of inflation the universe is hot enough to provide a smooth transition to the radiation era.

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I. INTRODUCTION

An inflationary phase gets rid of the major problems in cosmology, namely flatness, the horizon problem, homogeneity and numerical density of monopoles[1]. As strange as it may sound at first, it is actually very easy to have an exponential behavior of the scale factor. The main problem is how to attach the observed universe to the end of the inflationary epoch.

There are three possible solutions to this problem: reheating[1], preheating[2], and warm inflation[3, 4, 5, 6] (WI, from now on). In this work we focus on the latter, according to which the radiation is produced throughout inflation such that its energy density is kept nearly constant in this phase. This is accomplished by introducing a dissipation term, Γ , in the equation of motion for the inflaton, granting a continuous energy transfer from its decay. As a consequence, no reheating or preheating is necessary since we expect that enough radiation is produced to provide a smooth transition to the radiation era at the end of inflation.

On the other hand, being a two-field model, we are obliged to take into account entropy (isocurvature) perturbations[7, 8] if the problem of structure formation is considered. As is well known, the very existence of the former may provide non trivial evolution of the curvature perturbation on comoving hypersurfaces, \mathcal{R} , since this quantity is no longer frozen for large scale modes as in single field inflationary models. The usual procedure for determining the amplitude of the observed perturbations takes the fact that $\dot{\mathcal{R}} = 0$ for granted as soon as a given mode crosses outside the Hubble radius during inflation. The dissipation term is of central importance for the entropy perturbations, since in its absence the radiation is decoupled from the scalar field and therefore redshifted away together with its fluctuations during inflation. Then the overall scenario is reduced to a single field, where it is known that the entropy perturbations are neglected for long wavelength modes. In the presence of dissipation, radiation is rather produced than washed away and the effective fluid is a mixture of mutually interacting radiation and scalar field. The variation of its own effective equation of state is responsible for the entropy perturbations.

The main goal of this paper is to discuss the problem of perturbations in WI; the role of the dissipation term in the evolution of the entropy perturbation is exhibited after the development of a consistent gauge-invariant procedure.

The paper is organized as follows. In Section II the basic equations of WI are introduced together with a generalization of the slow-roll parameters that allows a better characterization of the slow-roll phase and its end. The mixture of radiation and scalar field can be described as an effective fluid with a well defined equation of state. In Section III a consistent gauge-invariant treatment is developed in which the connection between entropy perturbations and dissipation is established. The expressions for scalar adiabatic perturbations and its spectral index in WI are shown in Section IV. Section V is devoted to presenting two models in WI, both in the weak dissipation regime. In this case the entropy perturbations can be neglected, and an estimate of the magnitude of the dissipation term was obtained, together with other relevant parameters of the model, after taking into account the COBE data. Although the smallness of the dissipation, the temperature at the end of inflation was calculated and shown to be reasonable for a smooth transition to the radiation era. Finally, in Section VI we conclude and trace out some perspectives of the present work.

II. DYNAMICAL ASPECTS OF WARM INFLATION AND THE GENERALIZED SLOW-ROLL PARAMETERS

Berera and Fang[3] considered flat FRW models endowed with interacting radiation and scalar field whose dynamics is governed by the Friedmann equation

$$H^2 = \frac{8\pi}{3m_{pl}^2} \left(\rho_r + \frac{1}{2}\dot{\phi}^2 + V(\phi) \right), \quad (1)$$

by the equation of motion of an homogeneous scalar field $\phi(t)$ in the effective potential $V(\phi)$

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'[\phi(t)] = 0, \quad (2)$$

and by the 1st Law of Thermodynamics

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2. \quad (3)$$

In these equations $H = \frac{\dot{a}}{a}$ is the Hubble parameter, m_{pl} is the Planck mass, dot and prime denote derivative with respect to the cosmological time t and the scalar field, respectively; ρ_r is the energy density of the radiation field. The friction term Γ is responsible for the decay of the scalar field into radiation.

Warm inflation is characterized by the accelerated growth of the scale factor driven by the potential term $V(\phi)$ that dominates over other energy terms in Eq. (1). During this phase: (i) the Friedmann equation becomes

$$H^2 \simeq \frac{8\pi}{3m_{pl}^2} V(\phi); \quad (4)$$

(ii) the $\ddot{\phi}$ term is neglected in Eq. (2), yielding

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H + \Gamma}; \quad (5)$$

(iii) and finally, since the scalar field steadily decays into radiation, it is assumed that $\dot{\rho}_r \approx 0$, or equivalently,

$$\rho_r \simeq \frac{\Gamma \dot{\phi}^2}{4H} \quad (6)$$

The inflationary regime is more precisely characterized by the introduction of the slow roll parameters. For the present scenario the generalization of the usual slow-roll parameters is given by

$$\epsilon_{wi} = \frac{3\dot{\phi}^2 + 4\rho_r}{2(V(\phi) + \frac{1}{2}\dot{\phi}^2 + \rho_r)} = \frac{m_{pl}^2}{4\pi(1+\alpha)} \left(\frac{H'(\phi)}{H(\phi)} \right)^2 - \frac{1}{3(1+\alpha)} \frac{H'\rho'_r}{H^3} \quad (7)$$

and

$$\eta_{wi} = -\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{m_{pl}^2}{4\pi(1+\alpha)} \frac{H''(\phi)}{H(\phi)} - \frac{\alpha'H}{(1+\alpha)H'} \epsilon_{wi} - \frac{1}{3(1+\alpha)H} \left(\frac{\rho'_r}{H} \right)', \quad (8)$$

where we have introduced $\alpha \equiv \frac{\Gamma}{3H}$ that can be either greater or less than the unity depending on the regime of strong or weak dissipation. Note that the above expressions are exact. The slow-roll phase is characterized by $\epsilon_{wi} \ll 1$, as a consequence of $V(\phi) \gg 1/2\dot{\phi}^2 + \rho_r$, which is necessary to validate Eq. (4), and by $\eta_{wi} \ll 1$ for the condition of neglecting the $\ddot{\phi}$ term in Eq. (2). Besides these conditions we must require $\rho'_r \simeq 0$ that stands for the quasi-stable production of radiation. During the slow-roll, these parameters can be expressed in terms of the usual slow-roll parameters ϵ and η of the super-cooled inflation as

$$\epsilon_{wi} \simeq \frac{\epsilon}{1+\alpha} \quad (9)$$

and

$$\eta_{wi} \simeq \frac{\eta}{1+\alpha} + \frac{\alpha'H}{(1+\alpha)^2 H'} \epsilon, \quad (10)$$

where $\epsilon \equiv \frac{m_{pl}^2}{4\pi} \left(\frac{H'(\phi)}{H(\phi)} \right)^2$ and $\eta \equiv \frac{m_{pl}^2}{4\pi} \left(\frac{H''(\phi)}{H(\phi)} \right)$ as usually defined[9].

The parameters ϵ_{wi} and η_{wi} are natural and direct generalizations of the parameters ϵ and η of the super-cooled inflation. In the absence of dissipation and radiation, the parameters defined in warm inflation reduce to the usual ones. Besides, the parameter ϵ_{wi} can also be understood as a direct measure of the effective equation of state relating the total pressure $p = \frac{1}{2}\dot{\phi}^2 - V(\phi) + p_r$ and the total energy $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_r$ through $p = -(1 - \frac{2}{3}\epsilon_{wi})\rho$. The definition of inflation as the period of accelerated expansion of the universe implies that $\frac{\ddot{a}}{a} > 0$, so that from the exact expression

$$\frac{\ddot{a}}{a} = (1 - \epsilon_{wi})H^2 \quad (11)$$

the inflationary epoch can be formally characterized by $\epsilon_{wi} < 1$. The end of inflation occurs for $\epsilon_{wi} = 1$, implying that, in this stage $\frac{1}{2}\dot{\phi}^2 + \rho_r$ is comparable to the potential $V(\phi)$, or equivalently $\left(\frac{1}{2}\dot{\phi}^2 + \rho_r\right)_{end} = V(\phi_e)$ (see Eq. (7)), where ϕ_e is the scalar field at the end of inflation. The condition $\dot{\rho}_r \simeq 0$ is valid in most of inflation, but it is violated at the beginning and at the end of inflation, as well.

The crucial aspect of warm inflation is that the temperature of radiation, T_r , must exceed the Hawking temperature, H , or

$$T_r \geq H, \quad (12)$$

during the inflationary expansion; the moment when $T_r = H$ is considered the beginning of warm inflation. This relation assures that thermal fluctuations of the scalar field dominate over the quantum fluctuations producing a distinct spectrum of primordial fluctuations from that of super-cooled inflation[3, 10].

Finally, we derive a very useful expression for the number of e-folds N associated to the scalar field. We set $a = a_e e^{N(\phi)}$, where $a_e = a(\phi_e)$, ϕ_e are the values of the scale factor and the scalar field at the end of inflation, N is the number of e-folds before the end of inflation. It can be easily found that, in the slow-roll approximation,

$$N = \pm \frac{4\pi}{m_{pl}^2} \int_{\phi_i}^{\phi_e} (1 + \alpha) \frac{H(\phi)}{H'(\phi)} d\phi \quad (13)$$

where the upper (lower) sign is used when $\dot{\phi} > 0 (< 0)$, so that N is always a positive quantity. Since we know that warm inflation is characterized by the dominance of the thermal component of radiation, it begins when $T_r = H$ at $\phi = \phi_*$, say. Then, denoting by N_{wi} the number of e-folds during the warm inflation, we have

$$N_{wi} = \pm \frac{4\pi}{m_{pl}^2} \int_{\phi_*}^{\phi_e} (1 + \alpha) \frac{H(\phi)}{H'(\phi)} d\phi. \quad (14)$$

In general, $\phi_* \neq \phi_i$ and ϕ_* depends on the typical parameters of the theory.

III. PERTURBATIONS IN WARM INFLATION

We consider the inhomogeneous perturbations of the FRW background described by the metric in the longitudinal gauge[7, 8]

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t)(1 - 2\Psi)\delta_{ij} dx^i dx^j, \quad (15)$$

where $a(t)$ is the scale factor, $\Phi = \Phi(t, \mathbf{x})$ and $\Psi = \Psi(t, \mathbf{x})$ are the metric perturbations. The background matter content is constituted by radiation and scalar field interacting through the friction term Γ as shown in the last Section. The spatial dependence of all perturbed quantities are of the form of plane waves $e^{i\mathbf{k}\cdot\mathbf{x}}$, k being the wave number, so that the perturbed field equations regarding now only their temporal parts (we omit the subscript k) are

$$\begin{aligned} -3H(H\Phi + \dot{\Phi}) - \frac{k^2}{a^2}\Phi &= \frac{4\pi}{m_{pl}^2}\delta\rho \\ \dot{\Phi} + H\Phi &= \frac{4\pi}{m_{pl}^2} \left(-\frac{4}{3k}\rho_r a v + \dot{\phi}_0 \delta\phi \right) \\ \ddot{\Phi} + 4H\dot{\Phi} + (2\dot{H} + 3H^2)\Phi &= \frac{4\pi}{m_{pl}^2}\delta p. \end{aligned} \quad (16)$$

In the above equations dot means derivative with respect to t , ρ_r and ϕ_0 are background energy density of radiation and the scalar field, respectively; $\delta\rho = \delta\rho_r + \dot{\phi}_0(\delta\phi) - \dot{\phi}_0^2\Phi + V'\delta\phi$ and $\delta p = \frac{1}{3}\delta\rho_r +$

$\dot{\phi}_0 (\delta\dot{\phi}) - \dot{\phi}_0^2 \Phi - V' \delta\phi$ are the perturbations of the total energy density and pressure, respectively; v originates from the decomposition of the velocity field as $\delta U_i = -\frac{ia k_i}{k} v e^{i\mathbf{k}\cdot\mathbf{x}}$ (see Bardeen[8]). Also, due to the fact that the perturbation of the total energy-momentum tensor does not give rise to anisotropic stress ($\delta T_j^i \propto \delta_j^i$), $\Psi = \Phi$. For the sake of completeness we present in the appendix the evolution equations for $\delta\phi$, $\delta\rho_r$ and v .

We assume that δp and $\delta\rho$ are connected by the relation

$$\delta p = c_s^2 \delta\rho + \tau \delta S \quad (17)$$

where $c_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{\dot{p}_r + \dot{p}_\phi}{\dot{\rho}_r + \dot{\rho}_\phi}$ is the effective velocity of sound of the fluid constituted by the mixture of radiation and scalar field. The quantity $\tau \delta S$ is the contribution to the perturbation of the pressure due to the variation of the effective equation of state that relates p and ρ , or the entropy perturbation. Introducing Eq. (17) into the third equation of the system (16), and taking into account the first equation, we obtain

$$\ddot{\Phi} + (4 + 3c_s^2) H \dot{\Phi} + [2\dot{H} + 3H^2 (1 + c_s^2)] \Phi + \frac{c_s^2 k^2}{a^2} \Phi = \frac{4\pi}{m_{pl}^2} \tau \delta S. \quad (18)$$

The rhs of the above equation accounts for the entropy perturbations. In the absence of such perturbations and for long wavelength perturbations, the gauge-invariant curvature perturbation on comoving hypersurfaces[7, 11]

$$\mathcal{R} = \frac{2(H\Phi + \dot{\Phi})}{3H(1+w)} + \Phi \quad (19)$$

with $w = \frac{p}{\rho} = \frac{p_r + p_\phi}{\rho_r + \rho_\phi}$, is conserved. In the absence of the scalar field $c_s^2 = w = \frac{1}{3}$, $\tau \delta S = 0$ (see also next expression). Writing $\rho = \rho_r + \frac{1}{2}\dot{\phi}^2 + V(\phi_0)$ and $p = \frac{1}{3}\rho_r + \frac{1}{2}\dot{\phi}^2 - V(\phi_0)$ in Eq. (17), yields

$$\tau \delta S = (1 - c_s^2) \delta\rho - \frac{2}{3} \delta\rho_r - 2V' \delta\phi \quad (20)$$

After some manipulation using the field equations (16), it results in

$$\tau \delta S = -\frac{m_{pl}^2}{4\pi} (H\Phi + \dot{\Phi}) A - \frac{m_{pl}^2(1 - c_s^2)}{4\pi a^2} k^2 \Phi - \frac{2}{3} \rho_r \left(\frac{4aV'v}{k\dot{\phi}_0} + \frac{\delta\rho_r}{\rho_r} \right) \quad (21)$$

where A is given by

$$A = \frac{8\rho_r (H + V'/\dot{\phi}_0) - 2\Gamma \dot{\phi}_0^2}{3(\dot{\phi}_0^2 + 4/3\rho_r)}. \quad (22)$$

Assuming slow-roll conditions (4)-(6), it can be shown that A is given by $A \approx -2\Gamma$, putting in evidence the role of dissipation in the entropy perturbations. In the case of $\Gamma = 0$ the radiation field decays exponentially during inflation implying that it can be neglected, and therefore the entropy perturbation term is reduced to

$$\tau \delta S = \frac{m_{pl}^2(1 - c_s^2)}{4\pi} \frac{k^2 \Phi}{a^2} = \frac{m_{pl}^2 V'}{6\pi H \dot{\phi}_0} \frac{k^2 \Phi}{a^2}, \quad (23)$$

which agrees with previous results[11] obtained for a single scalar field, and that vanishes for long wavelength perturbations. In this way, the dissipation term plays a crucial role in producing entropy perturbations by preventing the radiation field to be washed away during inflation. Therefore, during WI different matter components, scalar field and radiation, evolve such that eventually nonuniform spatial distributions but with uniform total energy density are present. This is, roughly speaking, the origin of the entropy perturbations.

Now, we can rewrite Eq. (19) a very useful form considering long wavelength perturbations. Using the comoving curvature perturbation \mathcal{R} , we have

$$\dot{\mathcal{R}} = -\frac{2(H\Phi + \dot{\Phi})A}{3H(1+w)} - \frac{16\pi\rho_r}{9m_{pl}^2(1+w)H} \left(\frac{4aV'v}{k\dot{\phi}_0} + \frac{\delta\rho_r}{\rho_r} \right) \quad (24)$$

which relates the change in the comoving curvature perturbation due to the source $\tau\delta S$. We can determine the dominant term of the rhs of the above equation during inflation (see appendix for details), that shows to be

$$\dot{\mathcal{R}} \simeq \frac{4\rho\Gamma}{3(\rho+p)} \Phi \quad (25)$$

As expected from the previous discussion, the entropy perturbation depends directly on the dissipation term. For very small dissipation, which is still allowed in warm inflation, the source term in the above equation can be neglected and the adiabatic perturbations will be mostly responsible for the primordial spectrum of inhomogeneities. Independently of all approximations used it is intuitive that the dissipation may play a central role in producing entropy perturbations, since in the absence of dissipation the radiation field is washed away during inflation and its effect in the overall scenario can be neglected. We remark that our result is in agreement with Taylor and Berera[12], although a gauge-invariant analysis were not carried out, and opposite to the one obtained by Wolung Lee and Fang[13].

In the last section we will consider a small dissipation such that Eq. (25) can be reduced to $\dot{\mathcal{R}} \simeq 0$, meaning that the entropy perturbations are no longer important and the primordial spectrum of perturbations is due only to adiabatic perturbations.

IV. SPECTRAL INDEX FOR SCALAR PERTURBATIONS

Based on the final discussion of the last section the spectrum of scalar adiabatic perturbations is given by

$$\delta_{ad} \simeq \frac{\delta\rho}{\rho+p} = \frac{3H}{\dot{\phi}_0} \delta\phi. \quad (26)$$

This expression must be evaluated for $k = aH$, that is when a given scale k crosses the Hubble radius. The fluctuations of the scalar field are now supposed to be of thermal origin instead of quantum fluctuations, or equivalently[3, 10]

$$(\delta\phi)^2 \simeq \frac{3H}{4\pi} T_r \quad (27)$$

where T_r is the temperature of the thermal bath. For quantum fluctuations we know that $(\delta\phi)_{quant}^2 \approx \frac{H^2}{4\pi}$, so that the dominance of thermal fluctuations is guaranteed for $T_r > H$ which occurs during the warm inflation. Despite the small dissipation, the spectrum of perturbations will depend strongly on the dissipation as we can see after substituting Eq. (27) into Eq. (26), and taking into account that

$$T_r = \left(\frac{30}{g\pi^2} \rho_r \right)^{1/4} \simeq \left(\frac{15}{2\pi^2 g} \frac{\Gamma\dot{\phi}_0^2}{H} \right)^{1/4} \quad (28)$$

which holds during the slow-roll, with $g \approx 100$ being the number of degrees of freedom for the radiation field. Then, we obtain

$$\delta_{ad}^2 \simeq 37.19 \left[\frac{\alpha^{1/6}(1+\alpha)}{H'} \right]^{3/2} \left(\frac{H}{m_{pl}} \right)^3 \quad (29)$$

It is also useful to describe this spectrum in terms of its spectral index, n_s , defined as

$$n_s - 1 = \frac{d \ln \delta_{ad}^2}{d \ln k}. \quad (30)$$

Using the usual assumptions (i) $a = e^{-N} a_e$ (see Section II); (ii) the fact that $k = a H$ for a given scale crossing the Hubble radius, one can show that

$$\frac{d \ln k}{d \phi_0} = \frac{4\pi}{m_{pl}^2} \frac{H}{H'} (1 + \alpha) (\epsilon_{wi} - 1). \quad (31)$$

For $\alpha = 0$, it follows that $\epsilon_{wi} = \epsilon$ and the expression established in super-cooled inflation is easily recovered[14]. Finally, considering Eqs. (29), (30) and (31) we found the following expression for n_s in terms of the slow-roll parameters

$$n_s = 1 - \frac{1}{2(1 - \epsilon + \alpha)} \left(\frac{(11 + 5\alpha)\epsilon}{2(1 + \alpha)} - 3\eta \right) - \frac{m_{pl}^2(1 + 7\alpha)\Gamma'H'}{48\pi\alpha(1 + \alpha)(1 - \epsilon + \alpha)H^2} \quad (32)$$

We remark that the above equation is general in the sense that no approximation regarding α was used, and there is a contribution of the variation of the dissipation term that can be relevant. For instance, let us consider the potential of the type $V(\phi) = \lambda^4 \phi^q$, and a constant dissipation Γ_0 . It follows that for $\alpha \ll 1$, $n_s \simeq 1 - \frac{1}{2} \left(\frac{11}{2}\epsilon - 3\eta \right)$, and for any q , n_s is always less than one as in super-cooled inflation. On the other hand, if $\alpha \gg 1$, $n_s \simeq 1 - \frac{1}{2\alpha} \left(\frac{5}{2}\epsilon - 3\eta \right)$, where one can easily show that only if $q > 12$ the index n_s can be greater than one giving rise to a blue spectrum. In the case of variable dissipation, we assume $\Gamma = \beta_2 \phi^2$ and the corresponding expression for the index n_s becomes $n_s \simeq 1 - \frac{1}{2(1 + \alpha - \epsilon)} \left(\frac{(11q + 4) + (5q + 28)\alpha}{2q(1 + \alpha)} \epsilon - 3\eta \right)$. In particular, if $q = 4$ this expression is reduced to $n_s \simeq 1 - \frac{3}{2(1 + \alpha - \epsilon)} (2\epsilon - \eta)$ which is always less than 1.

V. SOME WORKED EXAMPLES

In order to estimate consistently the role of dissipation during the warm inflation, let us consider the potential

$$V(\phi) = \lambda^4 \phi^4 \quad (33)$$

where λ is an adimensional parameter. According to ref. [4], the dissipation term can be written as $\Gamma = \beta_m \phi^m$, $m = 0, 2$. For the sake of simplicity we set $m = 2$ which yields

$$\alpha = \frac{m_{pl}\beta_2}{\sqrt{24\pi}\lambda^2}, \quad (34)$$

that is a constant, and $[\beta] = m_{pl}^{-1}$. During inflation Eq. (4) holds, and together with Eq. (5) the density of radiation given by Eq. (6) is

$$\rho_r \simeq \frac{m_{pl}^4 \lambda^4 \alpha}{2\pi(1 + \alpha)^2} \left(\frac{\phi}{m_{pl}} \right)^2. \quad (35)$$

The end of inflation is achieved when $\epsilon_{wi} = 1$, or

$$\left(\frac{\phi_e}{m_{pl}} \right)^2 \simeq \frac{1}{2\pi(1 + \alpha)} \left(1 + \frac{1}{\sqrt{1 + \alpha}} \right). \quad (36)$$

This expression is valid either for a regime of very strong ($\alpha \gg 1$), or very weak dissipation ($\alpha \ll 1$). For the first, we obtain $\left(\frac{\phi_e}{m_{pl}}\right)^2 \simeq \frac{1}{2\pi\alpha}$, whereas for the second situation $\left(\frac{\phi_e}{m_{pl}}\right)^2 \approx \frac{1}{\pi}$ as in the super-cooled inflation. The beginning of warm inflation is indicated by $\phi = \phi_*$ for which $T_r = H$. From Eqs. (4) and (28) it follows that

$$\left(\frac{\phi_*}{m_{pl}}\right)^2 \simeq \left(\frac{2.7\alpha}{128\pi^5(1+\alpha)^2\lambda^4}\right)^{1/3}. \quad (37)$$

The next equation is

$$N_{wi} = \frac{\pi(1+\alpha)}{m_{pl}^2}(\phi_*^2 - \phi_e^2) = 60, \quad (38)$$

obtained with Eqs. (4) and (14) and express the fact that the total number of e-folds must be (at least) of the order of 60 to solve the usual problems mentioned in the introduction. These last two equations provide an important constraint between the free parameters α and λ .

According to the discussion of Section III, we assume a small dissipation ($\alpha < 1$) such that we can neglect the contribution of the entropy perturbations and the adiabatic perturbations will be relevant for structure formation. However, we have no idea of how small actually α should be. A possible way of making an estimative of this parameter is to use the COBE-DMR[15] observation of the cosmic microwave radiation anisotropy, which asserts for the temperature anisotropy $\left(\frac{\Delta T}{T_0}\right)^2 \approx 3.43 \times 10^{-11}$, with $T_0 = 2.73K$. Then, following the usual assumptions[16] concerning the contribution of scalar perturbations to this measured quantity, we may have

$$\delta_{ad}^2 \simeq 3.43 \times 10^{-11}, \quad (39)$$

and this expression must be evaluated for a given scale crossing the Hubble radius 50 e-folds before the end of inflation. We also remark after determining the value of α we can verify if at the end of inflation there will be enough radiation necessary to the radiation era.

As in super-cooled inflation (see appendix) a scale of astrophysical interest crosses the Hubble radius at $N_{wi} = 50$. Thus, ϕ_{50} can be evaluated from Eq. (38), resulting

$$\left(\frac{\phi_{50}}{m_{pl}}\right)^2 = \frac{50}{\pi(1+\alpha)} + \left(\frac{\phi_e}{m_{pl}}\right)^2. \quad (40)$$

Now we are in conditions to determine the relevant parameters of the model, namely, λ , α , ϕ_* and ϕ_e . For this, we substitute Eqs. (33) and (34) into the expression for δ_{ad}^2 (Eq. (29)), and evaluate it at 50 e-folds before the end of inflation through Eq. (40). Finally, the last equation is obtained when we take the observational data of COBE-DMR on the cosmic microwave background anisotropy given by (39).

Therefore, solving the system formed by Eqs. (36), (37), (38) and (39), we find

$$\begin{aligned} \alpha &\simeq 1.04 \times 10^{-9}, \quad \lambda^4 \simeq 0.97 \times 10^{-17} \\ \phi_* &\simeq 4.40 m_{pl}, \quad \phi_e \simeq 0.56 m_{pl} \end{aligned} \quad (41)$$

The temperature at the end of inflation is evaluated from the amount of radiation at this epoch. Using the above parameters and Eq. (28), it is not difficult to arrive to the following temperature

$$T_{end} \simeq 6.28 \times 10^{-8} m_{pl}, \quad (42)$$

which is in good agreement with the temperature generated by reheating. These results are quite interesting. In Refs. [4, 10] the dissipation term is constrained in such a way to produce at the end of inflation a hot universe, and consequently a smooth transition to the radiation era. In these works previous estimations of the ratio between dissipation and the Hubble parameter can be of order of 10^{-7} or

less. Here, by considering consistently the production of adiabatic perturbations and constraining the amplitude of its spectrum with COBE data, we arrive to similar results. Finally, the spectral index of scalar perturbations can be calculated directly from Eq. (32), yielding

$$n_s \simeq 0.956 \quad (43)$$

that is slightly close to one than the corresponding obtained for super-cooled inflation that is found to be 0.941.

In this second and last example we choose the quadratic potential $V = \frac{1}{2}m^2\phi^2$ and a constant dissipation term, $\Gamma_0 = \text{constant}$. During inflation α is no longer constant given by

$$\alpha \simeq \frac{m_{pl}\Gamma_0}{\sqrt{12\pi m}\phi}. \quad (44)$$

The same steps adopted before will be followed straightforwardly in order to estimate the values of Γ_0 and m compatible with the assumption of small dissipation and the observational data. The radiation is assumed to be (see Eq. 4)

$$\rho_r \simeq \frac{\sqrt{3}\Gamma_0 m m_{pl}\phi}{96\pi^{3/2}} \left(\frac{\phi}{m_{pl}} + \frac{\Gamma_0}{\sqrt{12\pi m}} \right)^{-2} \quad (45)$$

from which the beginning of warm inflation can be calculated

$$\frac{2.7\Gamma_0}{16^2\pi^5\sqrt{12\pi m}} \left(\frac{\phi_*}{m_{pl}} + \frac{\Gamma_0}{\sqrt{12\pi m}} \right)^{-2} \simeq \left(\frac{m}{m_{pl}} \right)^2 \left(\frac{\phi_*}{m_{pl}} \right)^3. \quad (46)$$

The end of inflation occurs when $\epsilon_{wi} = 1$ or

$$\frac{1}{4\pi} \left(\frac{m_{pl}}{\phi_e} \right)^2 + \frac{\Gamma_0 m_{pl}^3}{128\sqrt{5}\pi^{3/2}m\phi_e^3} \left(\frac{\frac{\phi_e}{m_{pl}} - \frac{\Gamma_0}{\sqrt{12\pi m}}}{\frac{\phi_e}{m_{pl}} + \frac{\Gamma_0}{\sqrt{12\pi m}}} \right) \simeq \frac{m_{pl}}{\phi_e} \left(\frac{\phi_e}{m_{pl}} + \frac{\Gamma_0}{\sqrt{12\pi m}} \right). \quad (47)$$

The additional constraint comes from the imposition of $N_{wi} = 60$. then, Eq. (14) for this case is

$$N_{wi} = 2\pi \left[\left(\frac{\phi_*}{m_{pl}} + \frac{\Gamma_0}{\sqrt{12\pi m}} \right)^2 - \left(\frac{\phi_e}{m_{pl}} + \frac{\Gamma_0}{\sqrt{12\pi m}} \right)^2 \right] = 60 \quad (48)$$

The expression for ϕ_{50} is established from the above by setting $N_{wi} = 50$.

The last step is to express properly the spectrum of scalar perturbations for this specific case in the same way as done before. Then, after a direct calculation, taken into account the expression for ϕ_{50} and the observational constraint (Eq. (39)), we obtained

$$\begin{aligned} \Gamma_0 &\simeq 3.09 \times 10^{-16} m_{pl}, \quad m \simeq 5.5 \times 10^{-8} m_{pl} \\ \phi_* &\simeq 3.09 m_{pl}, \quad \phi_e \simeq 0.79 m_{pl} \end{aligned} \quad (49)$$

Despite the very tiny value of Γ_0 , it can be shown that, as in the last example, $\alpha \propto 10^{-9}$ during warm inflation. Finally, the temperature at the end of inflation is

$$T_{end} \simeq 0.52 \times 10^{-7} m_{pl} \quad (50)$$

which again is acceptable. The spectral index for adiabatic perturbations is $n_s \simeq 0.972$, whereas the corresponding for super-cooled inflation is $n_s \simeq 0.962$.

VI. CONCLUSIONS

In this paper we generalized the slow roll parameters applied in the usual super cooled inflation to the WI scenario. They are valid for any value of dissipation and can be used even if one wishes to allow time dependence of the energy density of radiation. Once we had well defined expansion parameters, we were able to study the evolution of the inflaton and radiation fields as well as of the entropy and adiabatic perturbations.

We showed that the entropy perturbation δS is proportional to the dissipation in the system. Since WI is feasible even for small values of Γ , we assumed that δS is negligible, and focused on the subsequent evolution of the adiabatic perturbations. The restrictions imposed upon them by the observational data worked as a consistency check — yielding small values for Γ — and provided the parameters (either λ or m), final temperature and spectral index n_s for the particular models chosen. Both presented a nearly scale-invariant spectrum, and temperatures comparable to the ones generated by the usual reheating mechanisms.

As next steps, we will check if different potentials broadly adopted in WI generate self-consistent pictures as above. We also intend to study the behavior of tensor perturbations in the same scenario.

VII. ACKNOWLEDGMENTS

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APPENDIX A: EVOLUTION EQUATIONS FOR $\delta\phi$, $\delta\rho_r$ AND v

The total energy momentum $T^\mu{}_\nu = T^\mu_{(r)\nu} + T^\mu_{(\phi)\nu}$, constituted by radiation and scalar field, is conserved, or $T^{\mu\nu}{}_{;\nu} = 0$. However, radiation and scalar field are in interaction (cf. Eqs. (2) and (3)), whose covariant description[13] is

$$T^\mu_{(r)\nu}{}_{;\mu} = -T^\mu_{(\phi)\nu}{}_{;\mu} = \Gamma(\phi, \alpha U^\alpha)\phi_{,\nu} \quad (\text{A1})$$

From the above equation we perturb the equation of energy balance of radiation and scalar field to derive the evolution laws for $\delta\rho_r$, $\delta\phi$, respectively, and also after perturbing the equation of motion for the velocity field we obtain the evolution of δU_α and consequently v . The calculations are tedious but direct and result in

$$\begin{aligned} (\delta\rho_r)^\cdot + 4H\delta\rho_r + \frac{4}{3}ka\rho_r v &= 4\rho_r\dot{\Phi} + \dot{\phi}_0^2\Gamma'\delta\phi + \Gamma\dot{\phi}_0(2(\delta\phi)^\cdot - 3\dot{\phi}_0\Phi) \\ (\delta\phi)^\cdot + (3H + \Gamma)(\delta\phi) + \left(\frac{k^2}{a^2} + V'' + \dot{\phi}_0\Gamma'\right)\delta\phi &= 4\dot{\phi}_0\dot{\Phi} + (\dot{\phi}_0\Gamma - 2V')\Phi \\ \dot{v} + \frac{\Gamma\dot{\phi}_0^2}{\rho_r}v + \frac{k}{a}\left(\Phi + \frac{\delta\rho_r}{4\rho_r} + \frac{3\Gamma\dot{\phi}_0}{4\rho_r}\delta\phi\right) &= 0 \end{aligned} \quad (\text{A2})$$

In order to obtain the leading term of the rhs of Eq. (24), we follow the approximation used in ref. [17]. Basically, it consists in neglecting $\dot{\Phi}$ and $(\delta\phi)^\cdot$ during inflation, and in addition we also neglect \dot{v} . Taking into account these approximations in the Eq. (24), we arrive to the following relation

$$\dot{\mathcal{R}} \simeq \frac{4\Gamma\rho}{3(\rho+p)} \left\{ \Phi + \frac{3\sqrt{\pi}\epsilon}{2} \frac{\delta\phi}{m_{pl}} - \frac{\epsilon}{12(1+\alpha)} \left[3\Phi + \left(\frac{3}{4} + \frac{1}{1+\alpha} \right) \frac{\delta\rho_r}{\rho_r} \right] \right\}. \quad (\text{A3})$$

Since during inflation we can set $\epsilon \ll 1$, we may approximate the above equation as given by Eq. (24). Nonetheless, even with a more rigorous calculation it becomes clear that the entropy perturbations depends directly on the dissipation.

APPENDIX B: MATCHING EQUATION IN WARM INFLATION

The matching equation in warm inflation is easily obtained using the same reasoning of standard super-cooled inflation. Then, a comoving scale k whose length $\lambda = \frac{1}{k}$ crosses the Hubble radius H^{-1} during inflation when $a_k \lambda_k = H^{-1}$. The matching equation relates k (or λ) with $N(k)$, the corresponding value of the number of e-folds before the end of inflation that this scale crosses the Hubble radius. We can write

$$\frac{k}{k_{today}} = \frac{a_k H_k}{a_{today} H_{today}} = \frac{a_k}{a_{end}} \frac{a_{end}}{a_{today}} \frac{H_k}{H_{today}} \quad (\text{B1})$$

where the subscript ‘today’ indicates the present values of the quantity and a_{end} is the scale factor at the end of inflation. Now, $a_k = a_{end} e^{-N(k)}$. Note above that there is no reference to reheating. As usual we assume adiabatic expansion since the end of inflation that implies $\frac{a_{end}}{a_{today}} \simeq \frac{2.73K}{T_{end}}$, where T_{end} is the temperature at the end of inflation, and its value is the same of the temperature generated at the reheating. After some direct calculation we arrive at

$$N(k) = 62 - \ln \left(\frac{k}{a_{today} H_{today}} \right) - \frac{1}{4} \ln \left(\frac{\rho_{end}}{V_k} \right) - \ln \left(\frac{10^{16} \text{ GeV}}{V_k^{1/4}} \right) \quad (\text{B2})$$

where $T_{end} = \rho_{end}^{1/4}$. Now, the scales of astrophysical interest today, say the size of a galaxy $\lambda \sim Mpc$ crosses, up to logarithm corrections, the outside the Hubble radius at $N_{gal} \sim 62 - 4 \ln 10 \sim 50$, where $H_{today}^{-1} \sim 10^4 Mpc$ and $a_{today} = 1$.

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